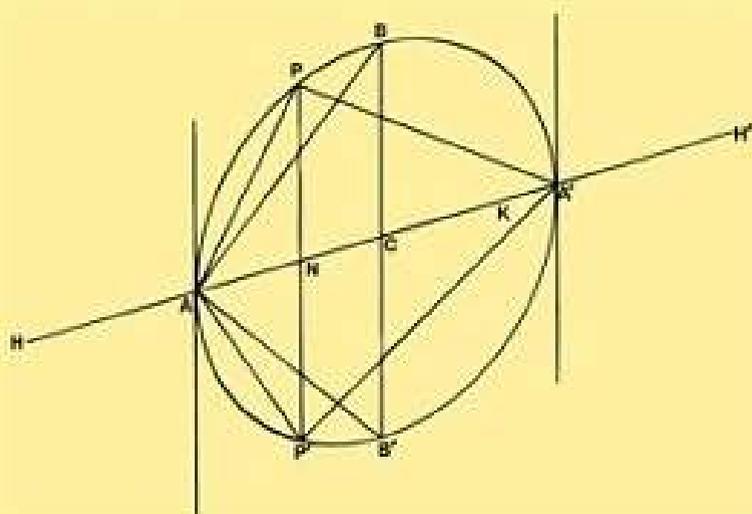


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THE WORKS OF ARCHIMEDES

EDITED IN MODERN NOTATION
WITH INTRODUCTORY CHAPTERS

EDITED BY THOMAS L. HEATH



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'It is not possible to find in geometry more difficult and troublesome questions, or more simple and lucid explanations. Some ascribe this to his natural genius; while others think that incredible effort and toil produced these, to all appearances, easy and unlaboured results.... His discoveries were numerous and admirable.' Plutarch on Archimedes in *The Parallel Lives*, c. 100 CE Archimedes lived in the third century BCE, and died in the siege of Syracuse. Together with Euclid and Apollonius, he was one of the three great mathematicians of the ancient world, credited with astonishing breadth of thought and brilliance of insight. His practical inventions included the water-screw for irrigation, catapults and grappling devices for military defence on land and sea, compound pulley systems for moving large masses, and a model for explaining solar eclipses. According to Plutarch, however, Archimedes viewed his mechanical inventions merely as 'diversions of geometry at play'. His principal focus lay in mathematics, where his achievements in geometry, arithmetic and mechanics included work on spheres, cylinders and floating objects. This classic 1897 text celebrated Archimedes' achievements. Part 1 placed Archimedes in his historical context and presented his mathematical methods and discoveries, while Part 2 contained translations of his complete known writings.

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The Works of Archimedes

*Edited in Modern Notation
with Introductory Chapters*

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THE WORKS
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EDITED IN MODERN NOTATION
WITH INTRODUCTORY CHAPTERS

BY

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PREFACE.

THIS book is intended to form a companion volume to my edition of the treatise of Apollonius on Conic Sections lately published. If it was worth while to attempt to make the work of "the great geometer" accessible to the mathematician of to-day who might not be able, in consequence of its length and of its form, either to read it in the original Greek or in a Latin translation, or, having read it, to master it and grasp the whole scheme of the treatise, I feel that I owe even less of an apology for offering to the public a reproduction, on the same lines, of the extant works of perhaps the greatest mathematical genius that the world has ever seen.

Michel Chasles has drawn an instructive distinction between the predominant features of the geometry of Archimedes and of the geometry which we find so highly developed in Apollonius. Their works may be regarded, says Chasles, as the origin and basis of two great inquiries which seem to share between them the domain of geometry. Apollonius is concerned with the *Geometry of Forms and Situations*, while in Archimedes we find the *Geometry of Measurements* dealing with the quadrature of curvilinear plane figures and with the quadrature and cubature of curved surfaces, investigations which "gave birth to the calculus of the infinite conceived and brought to perfection successively by Kepler, Cavalieri, Fermat, Leibniz, and Newton." But whether Archimedes is viewed as the man who, with the limited means at his disposal, nevertheless succeeded in performing what are really *integrations* for the purpose of finding the area of a parabolic segment and a

spiral, the surface and volume of a sphere and a segment of a sphere, and the volume of any segments of the solids of revolution of the second degree, whether he is seen finding the centre of gravity of a parabolic segment, calculating arithmetical approximations to the value of π , inventing a system for expressing in words any number up to that which we should write down with 1 followed by 80,000 billion ciphers, or inventing the whole science of hydrostatics and at the same time carrying it so far as to give a most complete investigation of the positions of rest and stability of a right segment of a paraboloid of revolution floating in a fluid, the intelligent reader cannot fail to be struck by the remarkable range of subjects and the mastery of treatment. And if these are such as to create genuine enthusiasm in the student of Archimedes, the style and method are no less irresistibly attractive. One feature which will probably most impress the mathematician accustomed to the rapidity and directness secured by the generality of modern methods is the *deliberation* with which Archimedes approaches the solution of any one of his main problems. Yet this very characteristic, with its incidental effects, is calculated to excite the more admiration because the method suggests the tactics of some great strategist who foresees everything; eliminates everything not immediately conducive to the execution of his plan, masters every position in its order, and then suddenly (when the very elaboration of the scheme has almost obscured, in the mind of the spectator, its ultimate object) strikes the final blow. Thus we read in Archimedes proposition after proposition the bearing of which is not immediately obvious but which we find infallibly used later on; and we are led on by such easy stages that the difficulty of the original problem, as presented at the outset, is scarcely appreciated. As Plutarch says, "it is not possible to find in geometry more difficult and troublesome questions, or more simple and lucid explanations." But it is decidedly a rhetorical exaggeration when Plutarch goes on to say that we are deceived

by the easiness of the successive steps into the belief that anyone could have discovered them for himself. On the contrary, the studied simplicity and the perfect finish of the treatises involve at the same time an element of mystery. Though each step depends upon the preceding ones, we are left in the dark as to how they were suggested to Archimedes. There is, in fact, much truth in a remark of Wallis to the effect that he seems "as it were of set purpose to have covered up the traces of his investigation as if he had grudged posterity the secret of his method of inquiry while he wished to extort from them assent to his results." Wallis adds with equal reason that not only Archimedes but nearly all the ancients so hid away from posterity their method of Analysis (though it is certain that they had one) that more modern mathematicians found it easier to invent a new Analysis than to seek out the old. This is no doubt the reason why Archimedes and other Greek geometers have received so little attention during the present century and why Archimedes is for the most part only vaguely remembered as the inventor of a screw, while even mathematicians scarcely know him except as the discoverer of the principle in hydrostatics which bears his name. It is only of recent years that we have had a satisfactory edition of the Greek text, that of Heiberg brought out in 1880-1, and I know of no complete translation since the German one of Nizze, published in 1824, which is now out of print and so rare that I had some difficulty in procuring a copy.

The plan of this work is then the same as that which I followed in editing the *Conics* of Apollonius. In this case, however, there has been less need as well as less opportunity for compression, and it has been possible to retain the numbering of the propositions and to enunciate them in a manner more nearly approaching the original without thereby making the enunciations obscure. Moreover, the subject matter is not so complicated as to necessitate absolute uniformity in the notation used (which is the only means whereby Apollonius can be made

even tolerably readable), though I have tried to secure as much uniformity as was fairly possible. My main object has been to present a perfectly faithful reproduction of the treatises as they have come down to us, neither adding anything nor leaving out anything essential or important. The notes are for the most part intended to throw light on particular points in the text or to supply proofs of propositions assumed by Archimedes as known; sometimes I have thought it right to insert within square brackets after certain propositions, and in the same type, notes designed to bring out the exact significance of those propositions, in cases where to place such notes in the Introduction or at the bottom of the page might lead to their being overlooked.

Much of the Introduction is, as will be seen, historical; the rest is devoted partly to giving a more general view of certain methods employed by Archimédès and of their mathematical significance than would be possible in notes to separate propositions, and partly to the discussion of certain questions arising out of the subject matter upon which we have no positive historical data to guide us. In these latter cases, where it is necessary to put forward hypotheses for the purpose of explaining obscure points, I have been careful to call attention to their speculative character, though I have given the historical evidence where such can be quoted in support of a particular hypothesis, my object being to place side by side the authentic information which we possess and the inferences which have been or may be drawn from it, in order that the reader may be in a position to judge for himself how far he can accept the latter as probable. Perhaps I may be thought to owe an apology for the length of one chapter on the so-called *νεύσεις*, or *inclinationes*, which goes somewhat beyond what is necessary for the elucidation of Archimedes; but the subject is interesting, and I thought it well to make my account of it as complete as possible in order to round off, as it were, my studies in Apollonius and Archimedes.

I have had one disappointment in preparing this book for the press. I was particularly anxious to place on or opposite the title-page a portrait of Archimedes, and I was encouraged in this idea by the fact that the title-page of Torelli's edition bears a representation in medallion form on which are endorsed the words *Archimedis effigies marmorea in veteri anaglypho Romae asservato*. Caution was however suggested when I found two more portraits wholly unlike this but still claiming to represent Archimedes, one of them appearing at the beginning of Peyrard's French translation of 1807, and the other in Gronovius' *Thesaurus Graecarum Antiquitatum*; and I thought it well to inquire further into the matter. I am now informed by Dr A. S. Murray of the British Museum that there does not appear to be any authority for any one of the three, and that writers on iconography apparently do not recognise an Archimedes among existing portraits. I was, therefore, reluctantly obliged to give up my idea.

The proof sheets have, as on the former occasion, been read over by my brother, Dr R. S. Heath, Principal of Mason College, Birmingham; and I desire to take this opportunity of thanking him for undertaking what might well have seemed, to any one less genuinely interested in Greek geometry, a thankless task.

T. L. HEATH.

March, 1897.

LIST OF THE PRINCIPAL WORKS CONSULTED.

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- C. A. BRETSCHNEIDER, *Die Geometrie und die Geometer vor Euklides.* (Leipzig, 1870.)
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INTRODUCTION.

CHAPTER I.

ARCHIMEDES.

A LIFE of Archimedes was written by one Heracleides*, but this biography has not survived, and such particulars as are known have to be collected from many various sources†. According to Tzetzes‡ he died at the age of 75, and, as he perished in the sack of Syracuse (B.C. 212), it follows that he was probably born about 287 B.C. He was the son of Pheidias the astronomer§, and was on intimate terms with, if not related to, king Hieron and his

* Eutocius mentions this work in his commentary on Archimedes' *Measurement of the circle*, ὡς φησιν Ἡρακλείδης ἐν τῷ Ἀρχιμήδους βίῳ. He alludes to it again in his commentary on Apollonius' *Conics* (ed. Heiberg, Vol. II. p. 168), where, however, the name is wrongly given as Ἡράκλειος. This Heracleides is perhaps the same as the Heracleides mentioned by Archimedes himself in the preface to his book *On Spirals*.

† An exhaustive collection of the materials is given in Heiberg's *Quaestiones Archimedeae* (1879). The preface to Torelli's edition also gives the main points, and the same work (pp. 363—370) quotes at length most of the original references to the mechanical inventions of Archimedes. Further, the article *Archimedes* (by Hultsch) in Pauly-Wissowa's *Real-Encyclopädie der classischen Altertumswissenschaften* gives an entirely admirable summary of all the available information. See also Susemihl's *Geschichte der griechischen Litteratur in der Alexandrinerzeit*, I. pp. 723—733.

‡ Tzetzes, *Chiliad.*, II. 35, 105.

§ Pheidias is mentioned in the *Sand-reckoner* of Archimedes, τῶν προτέρων ἀστρολόγων Εὐδόξου...Φειδία δὲ τοῦ ἀμοῦ πατρὸς (the last words being the correction of Blass for τοῦ Ἀκούπαρος, the reading of the text). Cf. Schol. Clark. in Gregor. Nazianz. Or. 34, p. 355 a Morel. Φειδίας τὸ μὲν γένος ἦν Συρακῶσιος ἀστρολόγος ὁ Ἀρχιμήδους πατήρ.